

# Probing anomalous $t\bar{t}Z$ interactions with rare meson decays

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## Abstract

Anomalous couplings of the  $Z$  boson to top quarks are only marginally constrained by direct searches and are still sensitive to new particle dynamics at the TeV scale. Employing an effective field theory approach we consider the dimension-six operators which generate deviations from the standard-model vector and axial-vector interactions. We show that rare  $B$  and  $K$  meson decays together with electroweak precision observables provide strong constraints on these couplings. We also consider constraints from  $t$ -channel single-top production.

## 1 Introduction

In the case of no direct observation of new particles at the Large Hadron Collider (LHC), precision measurements of the Standard Model (SM) interactions provide an alternative route to discover New Physics (NP). The measurement of the  $t\bar{t}Z$  production cross section at the LHC [1, 2] offers a direct test of anomalous  $t\bar{t}Z$  couplings. In a recent publication, Schulze and Röntsch presented the calculation and analysis of  $t\bar{t}Z$  production at next-to-leading order in QCD [3]. They used current and future-projected LHC data to constrain anomalous vector and axial-vector couplings of the  $Z$  boson to top quarks.

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In this work we study constraints on anomalous  $t\bar{t}Z$  couplings arising from electroweak precision observables (EWPO), rare  $B$  and  $K$  meson decays, and t-channel single-top production. The  $t\bar{t}Z$  couplings contribute to the rate of rare meson decays and to EWPO via radiative corrections. Thus, in the absence of contributions to these observables from other sources, rare meson decays and EWPO offer strong bounds on anomalous  $t\bar{t}Z$  couplings. In fact, we show that the indirect constraints exceed the direct bounds, even after the projected high-luminosity upgrade of the LHC [3].

We focus on the rare decays  $B_s \rightarrow \mu^+\mu^-$ ,  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  and  $K_L \rightarrow \pi^0\nu\bar{\nu}$ . These decays proceed via  $Z$ -penguin diagrams<sup>1</sup> which, in the SM, are dominated by the top-quark loop. This makes these processes especially sensitive to anomalous  $t\bar{t}Z$  couplings. For this reason, the current observation of the  $B_s \rightarrow \mu^+\mu^-$  rate already competes with constraints from EWPO and the expected future improvements in the measurements of rare meson decay rates are especially encouraging as they may provide the first evidence for anomalous  $t\bar{t}Z$  interactions.

The use of rare decays to obtain bounds on anomalous  $Z$  couplings has already been discussed in Refs. [4,5]. The authors derived constraints on  $b\bar{b}Z$  couplings by relating them to flavor-changing couplings within the minimal flavor violation (MFV) framework. In this work, we consider constraints arising from operator mixing, induced by electroweak loops, within an effective field theory (EFT) approach. This strategy has already been proposed in Refs. [6,7] for flavor-changing neutral currents (FCNCs) involving the top quark, as well as for constraining anomalous  $Wtb$  couplings [8,9].

The remainder of the paper is structured as follows: In Sec. 2 we define the appropriate EFT approach to study possible deviations in  $t\bar{t}Z$  couplings. In Sec. 3 we revisit the main constraints from EWPO and t-channel single-top production, and present our analysis of rare meson decays. In Sec. 4 we discuss the numerical results and conclude.

## 2 Effective field theory

Our starting point is the SM augmented by gauge-invariant dimension-six operators constructed out of the SM fields. This may be thought of as the effective field theory after integrating out NP at some scale  $\Lambda$  significantly larger than the electroweak scale. We write the effective Lagrangian as

$$\mathcal{L}^{\text{eff}} = \mathcal{L}^{\text{SM}} + \sum_k \frac{1}{\Lambda^2} C_k Q_k. \quad (2.1)$$

We are interested in the subset of operators that induce modifications of the vector and axial-vector  $t\bar{t}Z$  couplings at tree-level. They will have the main impact on direct searches for anomalous  $t\bar{t}Z$  couplings. If we assume no correlations between Wilson coefficients, a

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<sup>1</sup>The photon penguin does not contribute to  $B_s \rightarrow \mu^+\mu^-$  due to conservation of the vector current.

complete set of such operators is given by (cf. [10, 11]):

$$\begin{aligned}
Q_{\phi q,33}^{(3)} &\equiv (\phi^\dagger i \overleftrightarrow{D}_\mu^a \phi) (\bar{Q}_{L,3} \gamma^\mu \sigma^a Q_{L,3}), \\
Q_{\phi q,33}^{(1)} &\equiv (\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\bar{Q}_{L,3} \gamma^\mu Q_{L,3}), \\
Q_{\phi u,33} &\equiv (\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\bar{t}_R \gamma^\mu t_R).
\end{aligned} \tag{2.2}$$

These operators contain the Higgs doublet  $\phi$ , the left-handed third-generation quark doublet  $Q_{L,3}$ , and the right-handed top quark  $t_R$ . Moreover,  $\sigma^a$  are the Pauli matrices and  $D_\mu$  is the SM gauge-covariant derivative and we defined

$$\begin{aligned}
(\phi^\dagger i \overleftrightarrow{D}_\mu \phi) &= i\phi^\dagger (D_\mu \phi) - i(D_\mu \phi)^\dagger \phi, \\
(\phi^\dagger i \overleftrightarrow{D}_\mu^a \phi) &= i\phi^\dagger \sigma^a (D_\mu \phi) - i(D_\mu \phi)^\dagger \sigma^a \phi,
\end{aligned} \tag{2.3}$$

so that the operators are manifestly Hermitian. Therefore, all Wilson coefficients considered in this work are real.

In order to include in Eq. (2.2) all operators that induce  $t\bar{t}Z$  at tree-level, we have chosen the basis in which the up-type quark Yukawa matrix is diagonal. Accordingly, we have

$$Q_{L,3} \equiv \left[ \sum_j V_{3j} d_{L,j} \right]. \tag{2.4}$$

The fields  $t_{L(R)}$ ,  $d_{L,j}$  are mass eigenstates and  $V$  is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. A generic NP model can generate FCNC transitions in the up-quark sector; to take these effects into account we have to consider additional operators involving the first and second quark generation. However, in the restricted, but phenomenologically well-motivated, framework of MFV [12] these operators are suppressed with respect to those in Eq. (2.2) by elements of the CKM matrix. Thus, the resulting bounds on  $t\bar{t}Z$  couplings are negligible. Moreover, in the limit that only the top-quark Yukawa is non-vanishing in the MFV spurion counting, such additional operators are absent. For simplicity, we will use this approximation in the following, but we will comment on the effect of keeping a large bottom-quark Yukawa in Sec. 4. This discussion will cover MFV models with down-type quark alignment, i.e. models in which, at tree-level, only up-type quark FCNCs are generated by NP.

In our analysis, only the operators in Eq. (2.2) receive non-zero initial conditions at the scale  $\Lambda$ . However, electroweak corrections and corrections involving the SM top-quark Yukawa coupling  $y_t$  will induce mixing into the following additional operators relevant for our analysis:

$$\begin{aligned}
Q_{\phi q,ii}^{(3)} &\equiv (\phi^\dagger i \overleftrightarrow{D}_\mu^a \phi) (\bar{Q}_{L,i} \gamma^\mu \sigma^a Q_{L,i}), \\
Q_{\phi q,ii}^{(1)} &\equiv (\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\bar{Q}_{L,i} \gamma^\mu Q_{L,i}), \\
Q_{lq,33jj}^{(3)} &\equiv (\bar{Q}_{L,3} \gamma_\mu \sigma^a Q_{L,3}) (\bar{L}_{L,j} \gamma^\mu \sigma^a L_{L,j}), \\
Q_{lq,33jj}^{(1)} &\equiv (\bar{Q}_{L,3} \gamma_\mu Q_{L,3}) (\bar{L}_{L,j} \gamma^\mu L_{L,j}), \\
Q_{\phi D} &\equiv |\phi^\dagger D_\mu \phi|^2,
\end{aligned} \tag{2.5}$$

where  $i = 1, 2$  and  $j = 1, 2, 3$ . We can safely neglect the remaining Yukawa couplings in the mixing calculation. The operators in Eq. (2.5) will lead to additional bounds on the anomalous  $t\bar{t}Z$  couplings.

To study the effects of these operators on rare decays we need to know their Wilson coefficients at the electroweak scale where we match to the five-flavor effective theory. The running of the Wilson coefficients is given by the renormalization group equations (RGE). The matching is most conveniently performed in the broken phase where the Higgs field obtains its vacuum-expectation value,  $v$ . The relevant part of the effective Lagrangian in the broken phase reads

$$\mathcal{L}^{\text{eff}} \supset \mathcal{L}^{\text{SM}} + \frac{v^2}{\Lambda^2} (\mathcal{L}_Z^{\text{diag}} + \mathcal{L}_Z^{\text{FCNC}} + \mathcal{L}_W). \quad (2.6)$$

Using the conventions of Ref. [13], the flavor-diagonal neutral-current part of the Lagrangian is given by

$$\begin{aligned} \mathcal{L}_Z^{\text{diag}} = \frac{e}{2s_w c_w} Z_\mu \Big[ & (C_{\phi q, 33}^{(3)} - C_{\phi q, 33}^{(1)}) \bar{t}_L \gamma^\mu t_L - C_{\phi u, 33} \bar{t}_R \gamma^\mu t_R \\ & - \sum_{i=1,2,3} V_{i3}^* V_{i3} (C_{\phi q, ii}^{(3)} + C_{\phi q, ii}^{(1)}) \bar{b}_L \gamma^\mu b_L \Big]. \end{aligned} \quad (2.7)$$

This Lagrangian induces a shift of the SM  $Z$ -boson couplings to third-generation quarks. We follow the conventions in Ref. [14] and parametrize this shift by

$$\begin{aligned} \delta g_L^t &= \frac{v^2}{2\Lambda^2} (C_{\phi q, 33}^{(3)} - C_{\phi q, 33}^{(1)}), & \delta g_R^t &= -\frac{v^2}{2\Lambda^2} C_{\phi u, 33}, \\ \delta g_L^b &= -\frac{v^2}{2\Lambda^2} \sum_{i=1,2,3} V_{i3}^* V_{i3} (C_{\phi q, ii}^{(3)} + C_{\phi q, ii}^{(1)}). \end{aligned} \quad (2.8)$$

The operators  $Q_{\phi q, 33}^{(1)}$  and  $Q_{\phi u, 33}$  involve only neutral quark currents, whereas  $Q_{\phi q, 33}^{(3)}$  will necessarily induce deviations also in the effective  $W$  couplings. These charged-current contributions include

$$\mathcal{L}_W = \frac{e}{\sqrt{2}s_w} C_{\phi q, 33}^{(3)} \sum_i V_{3i} W_\mu^+ \bar{t}_L \gamma^\mu d_{L,i} + \text{h.c.} \quad (2.9)$$

The FCNC contributions relevant for us involve only down-type quarks and are given by

$$\mathcal{L}_Z^{\text{FCNC}} = -\frac{e}{2s_w c_w} (C_{\phi q, 33}^{(3)} + C_{\phi q, 33}^{(1)}) \sum_{i \neq j} V_{3i}^* V_{3j} Z_\mu \bar{d}_{L,i} \gamma^\mu d_{L,j}. \quad (2.10)$$

Note that Eq. (2.10) also receives contributions from the operators in Eq. (2.5), but these contributions cancel via the unitarity of the CKM matrix (see Sec. 3.3) and we do not include them here. Finally, we remark that the operators in Eq. (2.2) do not induce modifications of couplings involving the Higgs boson.

Constraints from flavor physics and EWPO suggest the absence of both tree-level down-type FCNC transitions and modifications of the  $b\bar{b}Z$  couplings if NP enters at the TeV scale. Thus, we take  $C_{\phi q,33}^{(3)} + C_{\phi q,33}^{(1)} = 0$  at the scale  $\Lambda$ . This relation can be realized in models with additional vector-like quarks [15]. Moreover, these models do not generate four-fermion operators at tree level. Under these assumptions, rare meson decays and the effective  $b\bar{b}Z$  couplings are sensitive to anomalous  $t\bar{t}Z$  couplings generated by the operators in Eq. (2.2).

### 3 Phenomenology

#### 3.1 t-channel single top-quark production

The operator  $Q_{\phi q,33}^{(3)}$  induces a tree-level correction to the  $Wtb$  coupling given by Eq. (2.9). Both the ATLAS and the CMS collaborations have recently measured the t-channel single top-quark production cross section [16, 17], which constitutes a direct measurement of the  $Wtb$  coupling. In particular, they report the measured cross-section normalized with respect to the SM expectation,  $\sqrt{\sigma_{\text{t-ch}}(t)/\sigma_{\text{t-ch}}^{\text{theo}}(t)} \equiv R_\sigma$ . The measured values reported by the two collaborations are

$$R_\sigma = \begin{cases} 0.97 \pm 0.10 & [\text{ATLAS}] \\ 0.998 \pm 0.041 & [\text{CMS}] \end{cases} \quad (3.1)$$

where we have combined the experimental and theoretical errors in quadrature. In our analysis this ratio is given by

$$R_\sigma = 1 + v^2 C_{\phi q,33}^{(3)} / \Lambda^2. \quad (3.2)$$

Combining the measurements, we find

$$v^2 C_{\phi q,33}^{(3)} / \Lambda^2 = -0.006 \pm 0.038. \quad (3.3)$$

#### 3.2 Electroweak precision observables

We saw in Sec. 2 that it is phenomenologically reasonable to assume  $C_{\phi q,33}^{(3)} + C_{\phi q,33}^{(1)} = 0$  at the scale  $\Lambda$ . In particular, this implies  $\delta g_L^b(\Lambda) = 0$ . Operator mixing will reintroduce a non-zero  $\delta g_L^b$  at the electroweak scale. The relevant parts of the RGE are

$$\begin{aligned} \mu \frac{d}{d\mu} C_{\phi q,33}^{(3)} &= \frac{y_t^2}{16\pi^2} \left( 8C_{\phi q,33}^{(3)} - 3C_{\phi q,33}^{(1)} \right) - \frac{g_2^2}{16\pi^2} \frac{11}{3} C_{\phi q,33}^{(3)}, \\ \mu \frac{d}{d\mu} C_{\phi q,33}^{(1)} &= \frac{y_t^2}{16\pi^2} \left( -9C_{\phi q,33}^{(3)} + 10C_{\phi q,33}^{(1)} - C_{\phi u} \right) + \frac{g_1^2}{16\pi^2} \frac{1}{9} (5C_{\phi q,33}^{(1)} + 4C_{\phi u}), \\ \mu \frac{d}{d\mu} C_{\phi q,11}^{(3)} &= \mu \frac{d}{d\mu} C_{\phi q,22}^{(3)} = \frac{g_2^2}{16\pi^2} 2C_{\phi q,33}^{(3)}, \\ \mu \frac{d}{d\mu} C_{\phi q,11}^{(1)} &= \mu \frac{d}{d\mu} C_{\phi q,22}^{(1)} = \frac{g_1^2}{16\pi^2} \frac{2}{9} (C_{\phi q,33}^{(1)} + 2C_{\phi u}), \end{aligned} \quad (3.4)$$

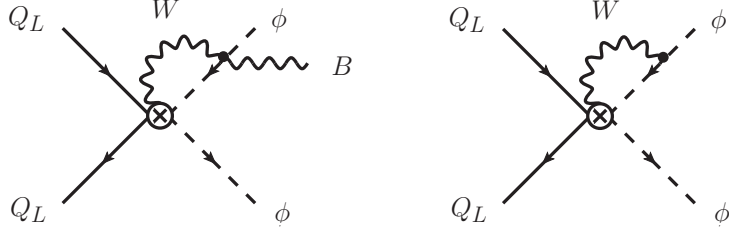


Figure 1: Diagrams that induce non-zero mixing proportional to gauge couplings of  $Q_{\phi q, 33}^{(3)}$  into itself. All other diagrams cancel among themselves or against contributions from the field renormalization. Note that there are additional contributions from penguin-type insertions. As a check, we have calculated the mixing both for external states including a  $B$  gauge boson (left) and for external states not including a  $B$  gauge boson (right) and find the same result.

with  $y_t$ ,  $g_1$ , and  $g_2$  the SM top-Yukawa coupling and the electroweak gauge couplings in the conventions of Ref. [13], respectively. The terms proportional to the top-quark Yukawa coupling have been presented in Ref. [18] and those proportional to the gauge couplings in Ref. [19]. However, our result for the latter differs from the one of Ref. [19] in the last term of the first line in Eq. (3.4). We have calculated the mixing in a general  $R_\xi$  gauge and find a gauge-independent result. While we agree with Ref. [19] on the terms arising from penguin-type insertions of the operators, we find an additional contribution for the mixing of  $Q_{\phi q, 33}^{(3)}$  into itself. It arises from the diagrams shown in Figure 1. An additional non-trivial check of our result can be found in the next section<sup>2</sup>.

Solving the RGE in Eqs. (3.4), we determine the Wilson coefficients at the electroweak scale:

$$\begin{aligned}
C_{\phi q, 33}^{(3)}(\mu_W) &= C_{\phi q, 33}^{(3)}(\Lambda) + \left\{ \frac{y_t^2}{16\pi^2} \left[ 8C_{\phi q, 33}^{(3)}(\Lambda) - 3C_{\phi q, 33}^{(1)}(\Lambda) \right] - \frac{g_2^2}{16\pi^2} \frac{11}{3} C_{\phi q, 33}^{(3)}(\Lambda) \right\} \log \frac{\mu_W}{\Lambda}, \\
C_{\phi q, 33}^{(1)}(\mu_W) &= C_{\phi q, 33}^{(1)}(\Lambda) + \left\{ \frac{y_t^2}{16\pi^2} \left[ 10C_{\phi q, 33}^{(1)}(\Lambda) - 9C_{\phi q, 33}^{(3)}(\Lambda) - C_{\phi u}(\Lambda) \right] \right. \\
&\quad \left. + \frac{g_1^2}{16\pi^2} \frac{1}{9} \left[ 5C_{\phi q, 33}^{(1)}(\Lambda) + 4C_{\phi u}(\Lambda) \right] \right\} \log \frac{\mu_W}{\Lambda}, \\
C_{\phi q, 11}^{(3)}(\mu_W) &= C_{\phi q, 22}^{(3)}(\mu_W) = \frac{g_2^2}{8\pi^2} C_{\phi q, 33}^{(3)}(\Lambda) \log \frac{\mu_W}{\Lambda}, \\
C_{\phi q, 11}^{(1)}(\mu_W) &= C_{\phi q, 22}^{(1)}(\mu_W) = \frac{g_1^2}{16\pi^2} \frac{2}{9} \left[ C_{\phi q, 33}^{(1)}(\Lambda) + 2C_{\phi u}(\Lambda) \right] \log \frac{\mu_W}{\Lambda}.
\end{aligned} \tag{3.5}$$

We used the leading-log approximation to relate the Wilson coefficient at the scale  $\Lambda$  to

<sup>2</sup>The authors of Ref. [19] confirmed that this contribution was accidentally omitted in their result; however, a corresponding term appears in their anomalous dimension of  $Q_{\phi l}^{(3)}$ .

the electroweak scale  $\mu_W$ . Using Eq. (2.8), we find

$$\begin{aligned} \delta g_L^b = & -\frac{v^2}{2\Lambda^2} \frac{\alpha}{4\pi} \left\{ V_{33}^* V_{33} \left[ \frac{x_t}{2s_w^2} \left( 8C_{\phi q,33}^{(1)}(\Lambda) - C_{\phi u}(\Lambda) \right) + \frac{17c_w^2 + s_w^2}{3s_w^2 c_w^2} C_{\phi q,33}^{(1)}(\Lambda) \right] \right. \\ & \left. + \left[ \frac{2s_w^2 - 18c_w^2}{9s_w^2 c_w^2} C_{\phi q,33}^{(1)}(\Lambda) + \frac{4}{9c_w^2} C_{\phi u}(\Lambda) \right] \right\} \log \frac{\mu_W}{\Lambda}. \end{aligned} \quad (3.6)$$

Here, we defined  $x_t \equiv m_t^2/M_W^2$  and used the relation  $C_{\phi q,33}^{(3)}(\Lambda) + C_{\phi q,33}^{(1)}(\Lambda) = 0$ .

The above corrections lead to a shift in the parameter  $\epsilon_b$ , defined in Ref. [20], given by  $\delta\epsilon_b = \delta g_L^b$ . The authors of Ref. [3] have compared their direct constraints on anomalous  $t\bar{t}Z$  couplings with the indirect constraint derived from  $\delta\epsilon_b$ . They use the expression for  $\delta\epsilon_b$  given in Ref. [21] while using the same effective operators as in our work. However, the calculation in Ref. [21] has been carried out in a different framework, namely, the non-linearly realized electroweak chiral Lagrangian [22]. Their result therefore does not agree with ours, and the expression for  $\delta\epsilon_b$  in Ref. [3] should be replaced by the one in Eq. (3.6).

Quantum corrections also induce the mixing of the operators in Eq. (2.2) into  $Q_{\phi D}$  in Eq. (2.5). This operator is tightly constrained by EWPO, since it leads to the universal oblique  $T$  parameter [23–25]. The mixing is given by [18, 19]

$$16\pi^2 \mu \frac{d}{d\mu} C_{\phi D} = \frac{8}{3} g_1^2 \left( C_{\phi q,33}^{(1)} + 2C_{\phi u,33} \right) + 24y_t^2 \left( C_{\phi q,33}^{(1)} - C_{\phi u,33} \right). \quad (3.7)$$

Accordingly, we obtain the following expression for the  $T$  parameter (cf. Ref. [26])

$$\begin{aligned} T = & -\frac{v^2}{2\alpha\Lambda^2} C_{\phi D} \\ = & -\frac{v^2}{\Lambda^2} \left[ \frac{1}{3\pi c_w^2} \left( C_{\phi q,33}^{(1)} + 2C_{\phi u,33} \right) + \frac{3x_t}{2\pi s_w^2} \left( C_{\phi q,33}^{(1)} - C_{\phi u,33} \right) \right] \log \frac{\mu_W}{\Lambda}. \end{aligned} \quad (3.8)$$

We checked that the  $T$  parameter obtained via the above RGE analysis agrees with a direct computation of the vacuum-polarization diagrams.

The  $T$  parameter is directly related to the quantity  $\epsilon_1$ , defined in Ref. [27], via  $\epsilon_1 \equiv \alpha T$ . The term proportional to  $x_t$  in Eq. (3.8) can be deduced from the corresponding result for  $\delta\epsilon_1$  in Ref. [28]; we agree with that result. However, our result disagrees with that quoted in Ref. [3], where the contributions from the modified charged current have not been included.

Note that there are no contributions to the  $S$  parameter within our setup because the operators in Eq.(2.2) do not mix into  $Q_{\phi WB} \equiv (\phi^\dagger \sigma^a \phi) W_{\mu\nu}^a B^{\mu\nu}$ . Vertex corrections could, however, lead to an indirect contribution to  $S$  (see e.g. Ref. [29]). This is illustrated, for instance, by the correlation of  $\delta\epsilon_3$  and  $\delta\epsilon_b$  [30]. We neglect these effects for simplicity. On the other hand, the operators in Eq.(2.2) mix into  $Q_{\phi l,11}^{(3)} \equiv (\phi^\dagger i \overleftrightarrow{D}_\mu^a \phi) (\bar{L}_{L,1} \gamma^\mu \sigma^a L_{L,1})$  according to

$$\mu \frac{d}{d\mu} C_{\phi l,11}^{(3)} = 2 \frac{g_2^2}{16\pi^2} C_{\phi q,33}^{(3)}. \quad (3.9)$$

This will induce a shift in the  $Z$  coupling to electrons, defined by

$$\delta g_L^e = -\frac{v^2}{2\Lambda^2} (C_{\phi l,11}^{(3)} + C_{\phi l,11}^{(1)}) . \quad (3.10)$$

Solving Eq. (3.9) we find

$$\delta g_L^e = -\frac{v^2}{\Lambda^2} \frac{\alpha}{4\pi s_w^2} C_{\phi q,33}^{(3)}(\Lambda) \log \frac{\mu_W}{\Lambda} . \quad (3.11)$$

Comparing with LEP data [31] we see that the resulting bound on  $C_{\phi q,33}^{(3)}$  is weaker than that from  $\delta g_L^b$  and we do not consider this constraint further. Similar considerations apply also for the corresponding  $\mu$  and  $\tau$  couplings. A more comprehensive analysis taking all of these effects into account might be worthwhile.

### 3.3 Rare meson decays

We now advocate the use of rare meson decays to constrain anomalous  $t\bar{t}Z$  couplings. Recall that in this work we assume  $C_{\phi q,33}^{(3)} + C_{\phi q,33}^{(1)} = 0$ , and thus the absence of tree-level FCNC transitions at the scale  $\Lambda$ . Operator mixing reintroduces these transitions at lower scales. We calculate the running of the Wilson coefficients from  $\Lambda$  to the electroweak scale, where we match onto the five-flavor effective theory. We then compute the modifications of the rare meson decay rates, which allows us to bound the Wilson coefficients at the scale  $\Lambda$ . We focus here on the processes  $B_{(s,d)} \rightarrow \mu^+ \mu^-$  and  $K \rightarrow \pi \nu \bar{\nu}$ . The reason is that all these decays are dominated by the  $Z$ -penguin within the SM and are thus the best candidates to constrain anomalous  $t\bar{t}Z$  couplings.

The part of the RGE relevant for the rare meson decays is the same as in Eq. (3.4), with the addition

$$\mu \frac{d}{d\mu} C_{lq}^{(3)} = -\frac{g_2^2}{16\pi^2} \frac{1}{3} C_{\phi q,33}^{(3)} , \quad \mu \frac{d}{d\mu} C_{lq}^{(1)} = \frac{g_1^2}{16\pi^2} \frac{1}{3} C_{\phi q,33}^{(1)} . \quad (3.12)$$

As a non-trivial check of our calculation we reproduce the logarithmic part of the loop functions  $f_{\nu\bar{\nu}}^{(j)}$  given in Ref. [9]. Solving the equations (3.12), we find the following expressions for the Wilson coefficients at the electroweak scale:

$$\begin{aligned} C_{lq}^{(3)}(\mu_W) &= C_{lq}^{(3)}(\Lambda) + \frac{1}{3} C_{\phi q,33}^{(3)}(\Lambda) \frac{g_2^2}{16\pi^2} \log \frac{\mu_W}{\Lambda} , \\ C_{lq}^{(1)}(\mu_W) &= C_{lq}^{(1)}(\Lambda) - \frac{1}{3} C_{\phi q,33}^{(1)}(\Lambda) \frac{g_1^2}{16\pi^2} \log \frac{\mu_W}{\Lambda} . \end{aligned} \quad (3.13)$$

The other relevant Wilson coefficients are given in Eq. (3.5). Note, however, that now some of the contributions to the FCNC transitions in Eq. (3.5) cancel because of the unitarity of the CKM matrix. In particular,  $C_{\phi q,ii}^{(1)}$  and  $C_{\phi q,ii}^{(3)}$  for  $i = 1, 2$  do not appear in the final expressions.



The rate of the rare decay  $B_s \rightarrow \mu^+ \mu^-$  was measured by the CMS [32] and LHCb [33] collaborations at the LHC and it was found to be consistent with the SM expectations [34]. The experiments are expected to reach the sensitivity of observing the SM  $B_d \rightarrow \mu^+ \mu^-$  rate in the next LHC run. Within the SM and the restricted set of operators we consider, a single operator  $Q_A \equiv (\bar{b}\gamma_\mu\gamma_5 q)(\bar{\mu}\gamma_5\mu)$  with  $q = d, s$  mediates the decays to a very good approximation. The rates depend on a single hadronic quantity, the decay constant  $f_{B_q}$  of the meson. The average time-integrated branching ratios then read [34]:

$$\overline{\text{Br}}_{q\mu} = \frac{|N|^2 M_{B_q}^3 f_{B_q}^2}{8\pi \Gamma_H^q} \beta_{q\mu} r_{q\mu}^2 |C_A(\mu_b)|^2 + \mathcal{O}(\alpha_{em}), \quad (3.14)$$

with  $M_{B_q}$  the mass of the  $B$  meson,  $r_{q\mu} = 2m_\mu/M_{B_q}$ ,  $\beta_{q\mu} = \sqrt{1 - r_{q\mu}^2}$ ,  $N = V_{tb}^* V_{tq} G_F^2 M_W^2 / \pi^2$  and  $\Gamma_H^q$  the total width of the heavy mass eigenstate in  $B_q$ - $\bar{B}_q$  mixing. In general, the time-integrated rates depend on the details of  $B_q$ - $\bar{B}_q$  mixing [35], but Eq. (3.14) holds to a very good approximation for both the  $B_s$  and the  $B_d$  system within the SM.  $C_A(\mu_b)$  is the Wilson coefficient of  $Q_A$ . It incorporates effects from heavy NP to the SM contribution and is evolved to the scale of the  $B$ -meson,  $\mu_b \sim m_b$ . The SM part includes perturbative next-to-leading-order (NLO) QCD [36–38] as well as recently calculated next-to-next-to-leading (NNLO) QCD [39] and NLO electroweak [40] corrections, which we all include in our analysis. However, at leading order, we have that  $C_A = -2(Y_0(x_t) + \delta Y^{\text{NP}})$  where  $Y_0(x_t)$  is the SM loop function and  $\delta Y^{\text{NP}}$  is the additional contribution from NP.

Similarly, the branching ratio for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  can be written as [41–43]

$$\begin{aligned} \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma)) \\ = \kappa_+(1 + \Delta_{\text{EM}}) \left[ \left( \frac{\text{Im}\lambda_t}{\lambda^5} X_t \right)^2 + \left( \frac{\text{Re}\lambda_c}{\lambda} (P_c + \delta P_{c,u}) + \frac{\text{Re}\lambda_t}{\lambda^5} X_t \right)^2 \right], \end{aligned} \quad (3.15)$$

where  $\lambda \equiv |V_{us}|$  and  $\lambda_i \equiv V_{is}^* V_{id}$ . The leading contribution is contained in the function  $X_t$  which comprises the top-quark loops. It is known to NLO in QCD and electroweak interactions [36, 38, 42, 44]. The parameter  $P_c$  describes the short-distance contribution of the charm quark and has been calculated including NNLO QCD [45–47] and NLO electroweak corrections [48]. Other long-distance contributions are contained in the parameter  $\delta P_{c,u}$  [43]. The hadronic matrix element of the low-energy operator is parameterized by  $\kappa_+$ . This parameter is extracted from  $K_{\ell 3}$  data and known precisely, including long-distance QED radiative corrections ( $\Delta_{\text{EM}}$ ), and NLO and partially NNLO corrections in chiral perturbation theory [49, 50].

The branching ratio of the  $CP$ -violating neutral mode involves only the top-quark contribution and can be written as<sup>3</sup>

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_L \left( \frac{\text{Im}\lambda_t}{\lambda^5} X_t \right)^2. \quad (3.16)$$

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<sup>3</sup>In fact, in our numerics we take into account also the (small) contribution from indirect  $CP$  violation, proportional to  $\epsilon_K$  – see Ref. [51].

Observable	Value	Ref.
$T$	$0.08 \pm 0.07$	[14]
$\delta g_L^b$	$0.0016 \pm 0.0015$	[14]
$\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ [CMS]	$(3.0_{-0.9}^{+1.0}) \times 10^{-9}$	[32]
$\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ [LHCb]	$(2.9_{-1.0}^{+1.1}) \times 10^{-9}$	[33]
$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$(1.73_{-1.05}^{+1.15}) \times 10^{-10}$	[54]

Table 1: Numerical input values for our fit.

Again, the hadronic matrix element can be extracted from the  $K_{l3}$  decays, parametrized here by  $\kappa_L$  [49].

As in the  $B_q \rightarrow \mu^+ \mu^-$  decays, we include all NP effects as additional contributions to the SM top-quark function  $X_t = X_t^{\text{SM}} + \delta X^{\text{NP}}$ . At the order we consider, all NP effects originate from the modifications of the  $t\bar{t}Z$  coupling. Thus, these are the same for  $B_q \rightarrow \mu^+ \mu^-$  and  $K \rightarrow \pi \nu \bar{\nu}$  decays

$$\delta Y^{\text{NP}} = \delta X^{\text{NP}} = \frac{x_t}{8} \left( C_{\phi u} - \frac{12 + 8x_t}{x_t} C_{\phi q, 33}^{(1)} \right) \frac{v^2}{\Lambda^2} \log \frac{\mu_W}{\Lambda}, \quad (3.17)$$

where  $x_t = m_t^2/M_W^2$  and we used again the relation  $C_{\phi q, 33}^{(3)} = -C_{\phi q, 33}^{(1)}$  at the scale  $\Lambda$ .

We conclude this section by comparing our work to existing results in the literature. Rare meson decays have been used to constrain FCNC  $Zqt$  couplings, where  $q = u, c$ , in Ref. [6, 7], and to constrain anomalous  $Wtb$  couplings in Ref. [9]. The corrections to the Wilson coefficients presented in the above publications amount to one-loop threshold corrections at the electroweak scale. These corrections are scheme dependent [52, 53]; the scheme dependence would cancel only when performing the two-loop running of the Wilson coefficients in the effective theory above the electroweak scale. On the other hand, the logarithmic dependence of the corrections on the scale  $\Lambda$  is scheme independent (since the leading-order anomalous dimensions are scheme independent). By effectively choosing a questionably low matching scale, of the order of the  $W$ -boson mass, these terms are rendered numerically insignificant in the above articles.

## 4 Numerics and discussion

In this section we present the constraints on anomalous  $t\bar{t}Z$  couplings derived from the observables discussed above. We show the individual constraints and perform a combined fit using current experimental data. In addition, we show the impact of future precision measurements of the rare  $B$  and  $K$  decay branching ratios.

Our main results are summarized in Fig. 2. In the left panel we show the individual 68% CL regions resulting from the measurement of the  $T$  parameter,  $\delta g_L^b$ ,  $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ ,

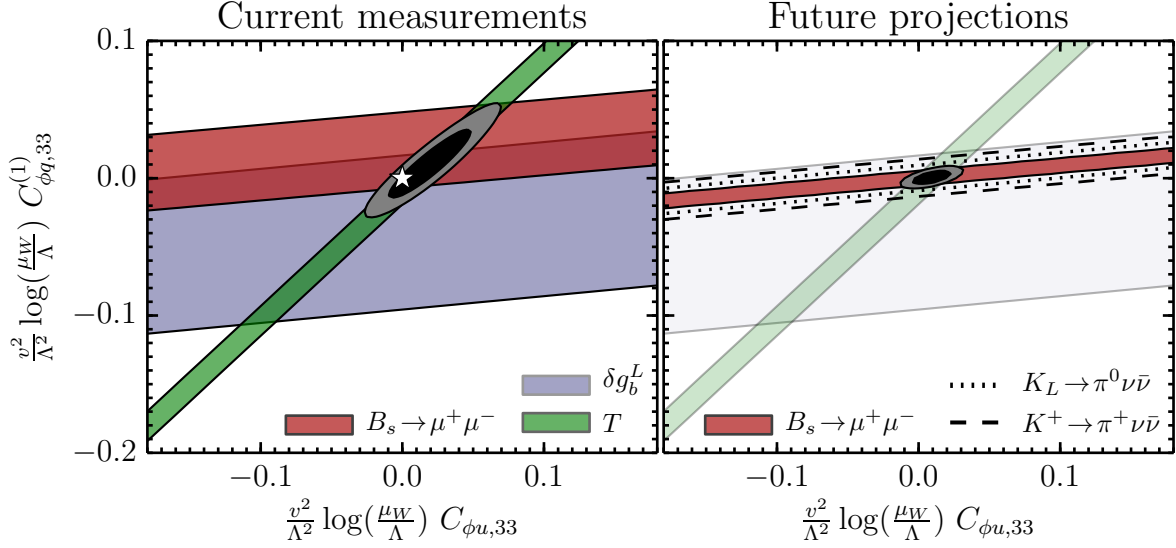


Figure 2: The preferred regions at 68% and 95% CL from our combined fit to EWPO and rare decays are shown as the dark-gray and light-gray ellipses, respectively. The colored bands show the 68% CL constraints from the individual observables. The star denotes the SM value.

and  $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ . In addition, we show the region compatible with the measurements in Table 1 at 68% and 95% CL.

We find that the branching ratio of  $B_s \rightarrow \mu^+ \mu^-$  and the  $T$  parameter currently lead to the most stringent constraints. In particular, the combination of the two leads to a strong bound on both Wilson coefficients  $C_{\phi q,33}^{(1)} \log(\mu_W/\Lambda) v^2/\Lambda^2$  and  $C_{\phi u,33} \log(\mu_W/\Lambda) v^2/\Lambda^2$ , of the order of a few percent. We note that t-channel single-top production leads to the bound  $-0.032 < v^2 C_{\phi q,33}^{(1)}/\Lambda^2 < 0.044$  (cf. Eq. (3.3)). This is weaker than the indirect bounds and leaves  $C_{\phi u,33}$  completely unconstrained.

In the future, we expect improvements in the measurement of  $\text{Br}(B_s \rightarrow \mu^+ \mu^-)$ , with a final uncertainty of  $\sim 5\%$  [55]. In addition, various experiments plan to measure the branching ratios of the rare  $K$  decays with high precision. The NA62 experiment at CERN aims at a final precision of  $\sim 10\%$  for the charged mode, which could be improved to  $\sim 3\%$  by an experiment at Fermilab [56]. The KOTO experiment aims at a similar precision for the neutral mode. On the other hand, the bounds from EWPO are mainly obtained from fits to LEP data and we do not expect any significant improvement within the next few years. In the right panel of Fig. 2 we show our future projections. As an illustration we assume a branching ratio measurement of all three rare decay modes with the SM central values and a precision of 5%. We keep the current constraints from the EWPO, but note that these bounds could be improved at future  $e^+e^-$  colliders [57].

The indirect constraints on the anomalous  $t\bar{t}Z$  couplings are much stronger than the constraints from direct searches, i.e. from  $t\bar{t} + Z$  production, even after a high-luminosity upgrade of the LHC. For instance, the authors of Ref. [3] give the bounds  $-0.04 <$

$v^2/\Lambda^2 C_{\phi q,33}^{(1)} < 0.19$  and  $-0.13 < v^2/\Lambda^2 C_{\phi u,33} < 0.32$ , assuming 3000 fb<sup>-1</sup> of data. However, one has to keep in mind that indirect constraints rely on a set of assumptions. In this work we assumed i) only  $C_{\phi q,33}^{(3)}$ ,  $C_{\phi q,33}^{(1)}$ , and  $C_{\phi u,33}$  receive non-zero initial conditions at the scale  $\Lambda$ ; ii)  $C_{\phi q,33}^{(3)} + C_{\phi q,33}^{(1)} = 0$  at  $\Lambda$ ; iii) only the top-quark Yukawa coupling is non-vanishing.

Assumption i) is compatible only with NP models with non-trivial flavor structure, while assumption ii) can be motivated by explicit models [15]. A simple way to deviate from assumption iii) is to consider models with a large enhancement of the bottom-quark Yukawa coupling; a generic example is a two Higgs-doublet model with large  $\tan\beta$ . The large bottom-Yukawa coupling will induce flavor off-diagonal versions of the operators in Eq. (2.2) and Eq. (2.5), via the MFV counting. These off-diagonal operators lead to additional contributions to FCNC top decays and  $D^0 - \bar{D}^0$  mixing. In order to relate these observables to  $t\bar{t}Z$  couplings, we assume MFV. Thus the resulting constraints on anomalous  $t\bar{t}Z$  couplings are suppressed by CKM-matrix elements. As an illustrative example consider an extreme case where the bottom-Yukawa coupling is much larger than the top-Yukawa coupling. In this case, we have  $C_{\phi q,23}^{(3)} \sim \lambda^2 C_{\phi q,33}^{(3)}$  etc., where  $\lambda \equiv |V_{us}| \approx 0.22$  is the Wolfenstein parameter. Then  $D^0 - \bar{D}^0$  mixing is suppressed by  $\lambda^{10} \approx 10^{-7}$  and thus completely negligible. Also, the branching ratio for  $t \rightarrow cZ$  is

$$\text{Br}(t \rightarrow cZ) \simeq \frac{\lambda^4 v^4}{\Lambda^4} \left[ \left( C_{\phi q,33}^{(3)} - C_{\phi q,33}^{(1)} \right)^2 + C_{\phi u,33}^2 \right]. \quad (4.1)$$

Using the present bound  $\text{Br}(t \rightarrow cZ) < 0.05\%$  given by the CMS collaboration [58] we see that the resulting bounds are not competitive with bounds from EWPO and rare  $B/K$  decays.

Note that the off-diagonal operators will also lead to additional contributions to rare  $B/K$  decays and anomalous  $b\bar{b}Z$  couplings. The generalization of our assumption ii) can be used to eliminate such contribution from these off-diagonal operators [15].

More generally, all rare decays in the down sector which receive a  $Z$ -penguin contribution can be used to obtain bounds on anomalous  $t\bar{t}Z$  couplings with our method. Suitable decays which will be measured in the future include  $B_d \rightarrow \mu^+ \mu^-$  [59] and  $B \rightarrow K \nu \bar{\nu}$  [60]. It would be interesting to allow for complex Wilson coefficients of the operators in Eq. (2.2) and study their effect on  $CP$  violation in rare meson decays.

To conclude, in this work we studied the effects of dimension-six operators, generating anomalous vector and axial-vector  $t\bar{t}Z$  couplings at tree-level, on precision observables. In particular, we advocate the use of rare  $K$  and  $B$  meson decays to obtain strong constraints on  $t\bar{t}Z$  couplings.

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